

MATH AND STATS LEARNING CENTRE MAGAZINE
Issue No. 3 Summer 2017

THE RESILIENCE

ISSUE



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NOTE FROM THE EDITOR

Each year it's a complete pleasure to put together the MSLC Magazine. The amazing opportunity to work—yet again—with the committee, the endless conversations we have as a team, the immense amount of hours that goes into it, the recruitment of contributors—it's all worth it!

The theme of this Summer 2017 Issue is Resiliency. We feel very strongly that this is a life skill; helpful not only in University but in our lives beyond as well. Through this issue, we hope to demonstrate the different faces of resiliency, let that be through MSLC TA reflections (pg.4) or one of our own student's journey through Statistics (pg.15). In many ways, this story is the story of us all. We all have struggled through one course or the other. Where some of us may differ is on how we recovered from that struggle. Did we show our resilience or did we let our fear get the better of us?

Resilience is the ability to thrive in light of adversity—it isn't about whether or not we crumble under pressure or that our grades falter as a result, but rather how we rise back from it—how we learn from our mistakes and use that to guide us in the future.

In this issue we explore resiliency within the field of Mathematics as well. We tackle some of the abstract concepts such as Infinity (pg.6), Golden Ratio (pg.10) and Symplectic Camel Theorem (pg.3). Surely, if it hadn't been for the historic work done by our resilient mathematical scholars, mathematics as we know it today wouldn't exist!

It is our hope that you do something this summer that helps you build up your *own* resilience. Enjoy some you-time and discover your strengths and areas that need improvement. Work on yourself, because that's the only way you could thrive and not just survive—and no one will do it for you!

I want to take this opportunity to thank the MSLC Magazine committee members for their contribution and hard work. Finally, a huge thanks to our Illustrator and all the contributors, without the work of whom this issue wouldn't have been possible.

Until next time,
 Manaal Hussain B.Ed., M.Ed.
 UTSC Alumni and Staff



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SYMPLECTIC CAMEL THEOREM

BY ZOHREH SHAHBAZI

MSLC EDITOR IN CHIEF, MATHEMATICS PROFESSOR

Symplectic manifolds are used to formulate classical mechanical systems. An important axiom in symplectic geometry tells us that a symplectic ball cannot be squeezed through a narrow cylinder or hole. This is called Symplectic Camel Theorem. The name comes from a story told in multiple religious texts. It is written that an arrogant person will never enter the garden (heaven) until a camel can pass through the eye of a needle--that is, never. In this short article, I will attempt to explain the Camel Theorem and its significance, and leave it to you to judge if it lives up to its name.

First, let's define what a manifold is. If you were to sew tiny pieces of a straight line such that they stick together smoothly, the result would be a one dimensional manifold. In other words, a one dimensional manifold is a curved line that is also smooth--there is no sharp point on its curve, nor is there a gap.

We can also define two dimensional manifolds as smooth surfaces that are made of tiny flat planes sewn together smoothly. The surface of a ball is an example of a two dimensional manifold.

We can define three dimensional manifolds as objects which locally possess the geometry of a ball, such that the balls are stitched together smoothly. For example, every time you bite into a doughnut, you are crushing a three dimensional manifold.

Furthermore, we can define n-dimensional manifolds. You might be inclined to ask what I mean by this. One example of the application of higher level manifolds is seen in Einstein's work when he describes space and time in terms of a four dimensional manifold, assigning three dimensions for location and one dimension for time. This four dimensional manifold model of space-time has proven to have considerable predictive power in explaining the natural world.



So what exactly is a *symplectic* manifold? A symplectic manifold is a manifold with an even number of dimensions, equipped with a mathematical structure to measure area, which could be used to study a moving object—a falling apple, for instance. For example, consider a system consisting of 9 particles that move smoothly in space. We can associate a 54-dimensional manifold with this system. Half of the dimensional elements describe the locations and the other half describe speeds in various directions of each of particles.

It has been proven by Gromov that a symplectic ball cannot be squeezed through a narrow cylinder or a hole. The arrogance and rigidity of the symplectic ball arises from the point where space and time are both entangled by the underlying manifold. That is why the ball cannot pass through a hole—in the same way that arrogant people must change their ways before the gates of heaven will open for them, a ball cannot pass through a hole while preserving its original structure.

Q&A WITH MSLC TAs

For this issue of the MSLC Magazine, we spoke to two of our Math and Statistics TA's about their experiences with the Centre—Fazle (Calculus TA) and Olivia (Statistics TA).

Fazle Chowdhury has been a TA at UTSC since the winter of 2008 and has been working under the CTL-MSLC as a Math TA since the summer of 2011. He has conducted over 55 tutorials, held over 650 Math help hours and over 25 review seminars. He shares tremendous passion and enthusiasm towards teaching and helping students in mathematics.

Olivia Rennie is a third year student currently pursuing a degree in Neuroscience, Psychology and Applied Statistics. She started her journey at MSLC in September 2015 as a Statistics Drop-in Facilitator; and this marks her fifth term here at MSLC. She worked her way up from a work-study position to a TA position.

Q. Tell us a bit about how you got

started in mathematics.

Fazle: My reasons for going the extra mile in mathematics developed during the second year of my undergraduate studies, when I was going through the worst depression while I was taking first year calculus! I still clearly remember my final mark in first year calculus, a 50% on the dot! In high school, calculus was one of my strongest subjects to the extent where I was actually fast tracking to complete my mathematics courses. Then something happened during first year—my calculus mark dropped from 91% in high school to 50% at U of T! I was obviously shocked, but I tried really hard to put together the best of my emotional and psychological effort to reason out, look back and ponder why I had performed so poorly.

Q. As Math and Stats TA's, you encounter students facing a wide

spectrum of issues with studying

math and statistics. What do you think could be improved about the way universities teach math and stats?

Olivia: I think a lot of the time students are used to lectures [in other disciplines] where they're just reading the material, and then they write the exam. In math, it's not like that. You need to be practicing, you need to be doing questions. Sometimes students don't even know how to use their calculators, which is painful. They have online homework and tutorials, but it's just not emphasized enough. They don't use their textbooks. They have a great textbook in STAB22 for example. Half of them don't even open it up or know it exists and that's a shame.

Fazle: There is something in students' minds that repeatedly tells them "Math is hard" or "OMG University level Math"! I'm still trying to figure

out the root cause of that. Before they even start classes they start aiming low, often without even attending the first lecture or tutorial! The majority of UTSC first year students are life science or management students taking first year calculus as a program requirement. They just want to pass and finish the course. I also find that it is difficult for them to focus or concentrate on a particular topic being taught in a lecture or explained in a tutorial. I believe the main reason for that is their immense engagement with electronic gadgets. I think students simply don't see the application of mathematics. They need to be walked through the applied or practical side of Mathematics and shown how it is used in real life. Also, many students don't know what the mathematical symbols they copy into their notes actually mean.

Q. Lastly, what do you like most about working at the MSLC?

Fazle: I have tremendous passion and enthusiasm for teaching and helping students in mathematics, and calculus in particular. I must express my gratitude to UTSC's CMS and CTL for providing me with this profound and highly valuable opportunity which has not only brought out my own mathematical teaching skills and passion, but also the ability to transfer these to the students and future ambassadors of mathematics. Year after year I've dealt with hundreds of students inside and outside of tutorials, before and after lectures, in math-aid centres, in the hallway, at Tim Hortons, at the UTSC bus stops and in the bus. I've never missed a chance to give them that extra bit of advice, suggestion, or guidance to perform better in mathematics regardless of how they were doing, as I believe they can always do better.

Mathematics is not about numbers, equations, computations, or algorithms: it is about UNDERSTANDING

- William Paul Thurston

Q. What do you think the MSLC could do to better serve students?

Olivia: It's amazing how fast the MSLC has been growing over the last few years, and the demand for a lot of our services has gone up considerably. I think that right now one of our focuses should be finding ways to offer more resources to students, both within the Centre, as well as outside of regular Centre hours. For instance, we offer online help for Math and Stats after hours, where students can ask TAs questions in a low-stress, convenient, and anonymous way. Right now, though, we only have a couple of hours per week available for this service, and so it would be great if we were able to expand this area. We are also very excited at the prospect of a larger Centre once the Highland Hall finishes construction, as having additional space will enable us to accommodate even more students in the room and potentially more TA support!

Olivia: I started at the Math and Stats Learning Centre at the beginning of second year, so still early in my university experience, and it really feels like it has become a part of me. It's not that I'm just here to make money, it's not that I'm just here to put something on my resume. I truly feel like it's become a part of me--a part of my identity. Getting to work at the MSLC has given me the opportunity to meet so many students I wouldn't have gotten the chance to meet. It's really opened my eyes to the diversity that exists here at UTSC, and it has opened my eyes to the different experiences students have and the things they struggle with. I've gotten to celebrate great grades--and not so great grades--but people open up about their academic journeys and where they want to go. It's just been a great experience. It's not just a TA position for me, it has made me who I am today, so I'm grateful for having the opportunity to work here.

A TINY GLIMPSE AT INFINITY

BY RAYMOND GRINNELL

ASSOCIATE PROFESSOR, TEACHING STREAM IN MATHEMATICS

Scientists estimate that there are at most 10^{19} grains of sand on earth and about 10^{80} atoms in the observable universe. These are big numbers indeed. It is fun to wrestle with even bigger things, like infinity and infinite sets. A lot of university mathematics is founded on infinite sets (e.g. calculus, linear algebra, and topology). Everyone probably has some idea of what infinity means: it is something that never stops, it has no end. Infinity and infinite sets are obviously really big. Aside from their bigness, these sets have some rather strange properties. Let's look at some ideas about infinite sets by starting with some finite ones.

Given a natural number n , let $I_n = \{1, 2, 3, \dots, n\}$. For example, we have $I_4 = \{1, 2, 3, 4\}$ and $I_8 = \{1, 2, 3, \dots, 7, 8\}$. Consider two more sets: $V = \{M, A, T, H\}$ and $W = \{C, O, M, P, U, T, E, R\}$. It is obvious that I_4 and V have the same number of elements and so do I_8 and W , but V and W do not. The reason why I_4 and V have the same number of elements is that they can be put in 1-1 correspondence with each other. This means there exists a function $f: I_4 \rightarrow V$ that is both 1-1 (i.e. each pair of different elements of I_4 gets mapped to different elements of V) and onto (i.e. every element of V is the output of f of some member of I_4). We call functions that are both 1-1 and onto "bijections". One example of a bijection f is $f(1) = M; f(2) = A; f(3) = T; f(4) = H$. Bijections are just formal 1-1 correspondence functions. It is easy to see that f is not unique and that there are bijections between I_8 and W . What is quite interesting is that there is no bijection between I_4 and W . This is because W has too many elements. There are plenty of 1-1 functions from I_4 into W . Here's one: $h: I_4 \rightarrow W$ given by $h(1) = C; h(2) = U; h(3) = T; h(4) = E$. Note that the elements O, M, P, R in W are not accounted for by the function h . It is not hard to imagine that

if h is any 1-1 function from I_4 to W , then it cannot ever be onto W .

Let X and Y be any nonempty sets. We say that X and Y have the same cardinality (and we write $\#X = \#Y$) to mean that there exists a bijection $f: X \rightarrow Y$. In other words, X and Y have the same cardinality exactly when there is a 1-1 correspondence between them. This means the elements of X can be paired-off in a 1-1, onto manner with the elements of Y . This is what the idea of sets having the same number of elements means. For our sets above, we see that I_4 and V have the same cardinality and so do I_8 and W , but V and W do not have the same cardinality. For arbitrary nonempty sets X and Y , we write $\#X < \#Y$ to mean that there exists a 1-1 function mapping X to Y but there is no onto function mapping X to Y . This definition makes precise the idea of one set having fewer elements than another. This is why we would write $\#V < \#W$. The function h above is 1-1, but there is no onto function from V to W . The set V has fewer elements than W , so we can never pair-off the elements of V with all elements of W , no matter how hard we try.

Let $N = \{1, 2, 3, 4, \dots\}$ and $S = \{1, 4, 9, 16, \dots\}$. We call N and S the set of natural numbers and squares, respectively. The elements of N and S can be put in 1-1 correspondence with each other: $1 \rightarrow 1, 2 \rightarrow 4, 3 \rightarrow 9, 4 \rightarrow 16$, and so on. We have a bijection $f: N \rightarrow S$ given by $f(n) = n^2$. Hence we write $\#N = \#S$ and we say that N and S have the same cardinality: they have the same number of elements. This is a subtle idea that sets can have the same number of elements even when that number of elements is not finite. If again X is an arbitrary set, we say that X is countably infinite to mean that $\#N = \#X$. In other words, X is countably infinite if and only if N and X can be put in 1-1 correspondence with each other.

Another way of saying this is that the elements of X can be enumerated $x_1, x_2, x_3, x_4, \dots$. This means there exists some listing of the elements of X in such a way that every element of X appears precisely once in the list. With these ideas in mind, it is now clear what we mean by a finite set. It is a set A that either has no elements or for which $\#A = \#I_n$ for some natural number n . A nonempty set A is finite when we can list its elements $a_1, a_2, a_3, \dots, a_n$, and stop. A finite set can have a gigantic number of elements, but it is still finite, so it is tiny when compared to any infinite set.

It seems strange that S is a much smaller subset of N , yet they have equal cardinality. But in the world of infinite sets, N and S have the same number of elements. This is exactly the subtle idea behind writing $\#X = \#Y$ for nonempty sets: they have the same number of elements, even when that number is some kind of infinity. Another important example is the following: Let P denote the set of positive rational numbers--that is, the set of fractions of natural numbers. It is obvious that every natural number is itself a positive rational number and that there are infinitely many elements of P that are not natural. We see that P is countably infinite because of the existence of the following interesting enumeration:

$$\frac{1}{1}, \frac{1}{2}, \frac{2}{1}, \frac{1}{3}, \frac{3}{1}, \frac{2}{3}, \frac{3}{2}, \frac{4}{1}, \dots, \dots, \dots$$

How do we invent this enumerating? We start with a 2-way infinite array with $1, 2, 3, 4, \dots$ across the top and down the left margin, and then form fractions. We enumerate in an increasingly diagonal fashion starting from the top-left, while omitting repeats:

	1	2	3	4	..
1	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$..
2	$\frac{2}{1}$	$\frac{2}{2}$	$\frac{2}{3}$	$\frac{2}{4}$..
3	$\frac{3}{1}$	$\frac{3}{2}$	$\frac{3}{3}$	$\frac{3}{4}$..
4	$\frac{4}{1}$	$\frac{4}{2}$	$\frac{4}{3}$	$\frac{4}{4}$	
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮

It is worthwhile to think for a moment that every positive rational fraction in lowest terms appears in the enumeration once and only once. Since N is a very small subset of P , it is almost unbelievable that they have the same number of elements.

The following ideas concerning infinity and infinite sets are even more bizarre. The set $(0, 1)$ consisting of all real numbers strictly between 0 and 1 can be shown to have a higher cardinality than N . We would write this as $\#N < \#(0, 1)$. This means no enumeration of $(0, 1)$ exists and that not all infinite sets are countably infinite. We might even say that the infinite set $(0, 1)$ is a more infinite set than the infinite set N . Next, given any two open intervals (a, b) and (c, d) there is a linear bijective function $I : (a, b) \rightarrow (c, d)$. We have $\#(a, b) = \#(c, d)$, yet the lengths of these intervals can be vastly different. This shows that our concept of cardinality and length are related but different. A stronger fact in this direction is that $\#(a, b) = \#R$, so (a, b) has the same number of real numbers as the set of all real numbers, R . There is a subtle mathematical proof sometimes given in mathematics courses like MATB43 or MATC37 that shows there are infinitely many infinite sets X_1, X_2, X_3, \dots such that $\#N < \#(0, 1) < \#X_1 < \#X_2 < \#X_3 \dots$. Thus there are infinitely many different infinities with $\#N$ being the smallest one.



DECAPITATED MATHEMATICS

BY BATUL FATIMA MIZRA

UTSC STUDENT



In Aristotle's typification of science (*dianoia*) and wisdom into the three divisions of theoretical, practical, and poetical, the first pertains to the macrocosm and objective world, while the second and third pertain to the microcosm and subjective world of humanity. To simplify, only the first two divisions are considered and the third, poetical wisdom, is subsumed in the second, practical wisdom. In traditional or sacred philosophy, such divisions are neither separative nor equal, and the classification of wisdom is always informed by unity and hierarchy. As such, theoretical wisdom is superordinate to practical wisdom and the subdivisions of both— metaphysics, mathematics, and physics in the case of the former, and ethics, domestics, and civics pertaining to the latter—are in turn hierarchically ordered. In what follows, we will consider only theoretical wisdom.

The highest theoretical wisdom, metaphysics, is the farthest from the empirical, since the principle and essence of metaphysics is the Absolute. Subordinate to metaphysics is mathematics and on the lowest level are the physical sciences (physics, chemistry, biology, etc.), which subsist by physical and sensorial experience and experimentation.

In the modern period, the hierarchy as outlined above was effectively truncated and flattened. It is as if modernity took a sword and cut through the neck of mathematics. The part that was left (the lower one), was a mere collection of procedures and calculations that in most cases are only useful when applied to other sciences, such as physics, and are used as a tool to achieve definite goals. The part that was removed, the higher part, is the one connected to higher metaphysical principles. Just as physics gains

structure and meaning only when taking its guidelines from the above (that is, from mathematics), mathematics is made meaningful only when it is inspired and connected to its superordinate science, metaphysics.

The reduction of the hierarchy to its inferior and lower aspects was itself a function of the general fall of humanity away from its Originating Principle and its becoming increasingly implicated in the realm of matter. Knowledge was now more about the purely quantitative aspect of matter and its particular and mutable nature, rather than quality and principles. Hence, the pole of action came to replace the pole of true and principled knowledge. In the traditional way of looking at the world, true knowledge was superior to action. Rene Guenon, the genius metaphysician and mathematician of the late 19th and early 20th centuries wrote in this regard,

"...just as the unchanging is superior to change. Action, being merely a transitory and momentary modification of the being, cannot possibly carry its principle and sufficient reason in itself; if it does not depend on a principle outside its own contingent domain, it is but illusion; and this principle, from which it draws all the reality it is capable of possessing—its existence and its very possibility can be found only in contemplation... Similarly change, in the widest sense of the word, is unintelligible and contradictory; in other words, it is impossible without a principle from which it proceeds and which, being its principle, cannot be subject to it, and is therefore necessarily unchanging; it was for this reason that, in the ancient world of the West, Aristotle asserted that there must be a 'unmoved mover' of all things"

When things cannot be known in their principle through a knowledge that derives from the Absolute, “knowledge” becomes a faint shadow of its true self, and a parody of it is produced such that much is said and written but little is truly understood.¹ Guenon in *The Metaphysical Principles of the Infinitesimal Calculus* speaks of such “conventionalism” as a malaise of modernity. He goes on to show how “conventionalism” and the ascription of conventional terms to real mathematical notions results in error, to such an extent that a logical science like mathematics becomes illogical. An example of such a convention is the notion of *infinity*. The true metaphysical meaning of infinity is that which properly and wholly pertains to the Absolute, while that which is called “infinity” today is a term full of self-contradiction. Therefore, Guenon suggests, that what modern mathematicians call “infinity” is actually “indefiniteness”. Guenon illustrates this by giving examples of the contradictions that have arisen when erroneous notions such as “the infinite number” are employed. He further illustrates this by showing how this notion was wrong from the beginning by examining the ideas of the “founder” of infinitesimal calculus, Leibnitz, who did not have a real sense of the notion.

When mathematics stays true to its real self, and is grounded in metaphysics, it is able to play its proper role as the isthmus and arbiter between the physical and the metaphysical worlds. There is then a mathematics that is not totally quantitative, but rather qualitative; one that

speaks of symbols and signs more than measurements and amounts. It is this type of mathematics that Pythagoras engaged in, the one that led him to the Mysteries that lie at the summit of human understanding and enlightenment. When mathematics loses this aspect, which functions as its source of legitimacy and its proverbial “head,” it spirals into a purely instrumental and meaningless technique that is used as a convention to describe and manipulate the physical and material world. A decapitated mathematics cannot survive for long and can only lead to an “order” that is chaotic and a “life” that is death itself.

References:

Gué non, R. (2001). *The Crisis of the Modern World*. Ghent, NY: Sophia Perennis.

Gué non, R. (2004). *The Metaphysical Principles of the Infinitesimal Calculus*. Hillsdale, NY: Sophia Perennis.

1. T.S. Eliot says much the same in *The Wasteland*. “What are the roots that clutch, what branches grow out of this stony rubbish? Son of man, you cannot say, or guess, for you know only a heap of broken images, where the sun beats, and the dead tree gives no shelter, the cricket no relief, and the dry stone no sound of water....”



THE GOLDEN RATIO

BY JIAHENG LI

UTSC STUDENT

& ZOHREH SHAHBAZI

ASSOCIATE PROFESSOR, TEACHING STREAM IN MATHEMATICS

The Milky Way is a spiral galaxy which contains our solar system. The diameter of The Milky Way is between 100,000 and 180,000 light years. It is estimated that the Milky Way contains about 100-400 billion stars. The entire galaxy rotates around the Milky Way's center at a rotational speed of 140 miles per second; the closer to the center, the higher the speed. Spiral-shaped galaxies like The Milky Way are common throughout the universe. Interestingly, the same type of spiral shape appears in many other natural phenomena. For example, peregrine falcons--the fastest birds on Earth--circle their prey in a spiral formation before they strike, allowing them to get a better view of their prey while maintaining maximum speed.



In a logarithmic spiral, the distance between each turn increases geometrically, while in an Archimedean spiral, the distance between each turn remains constant.

We can construct a logarithmic spiral from a golden triangle. But first, we must discuss the golden ratio and its connection with golden triangles.

Golden ratio is an irrational number defined as below:

$$\varphi = \frac{1 + \sqrt{5}}{2} = 1.618034 \dots$$

The golden ratio also satisfies the quadratic equation:

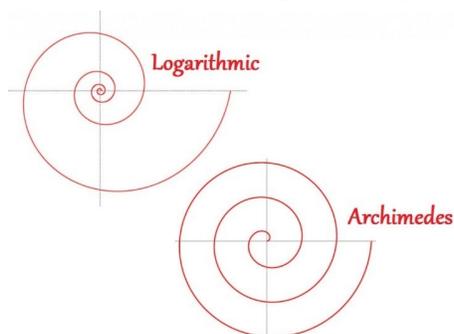
$$x^2 = 1 + x$$

The golden ratio appears throughout mathematics and has interesting properties.

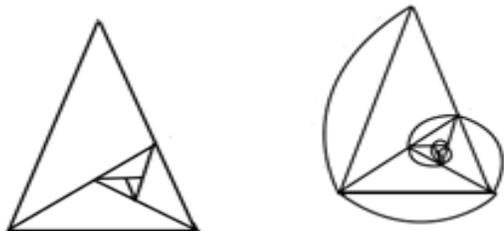
We can use the Fibonacci Sequence to express the golden ratio as a continued fraction. Recall that the first two terms are equal to 1, and that each subsequent term is the sum of the two previous terms. By comparing the ratios of two adjacent Fibonacci numbers, the golden ratio could be expressed by the following continued fraction:

$$\varphi = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$$

There are two types of spirals: logarithmic spirals and Archimedean spirals.



A golden triangle is an isosceles triangle where the ratio of its leg to its base is equal to the golden ratio. We can use a golden triangle to construct a logarithmic spiral by bisecting one of the base angles. The resulting triangle is still a golden triangle. We could continue this process ad infinitum, hypothetically producing an infinite number of golden triangles.



If we connect the vertices of all of those golden triangles, we will obtain a logarithmic spiral.



STATISTICS BRAIN TWISTERS!

Here are two Statistic Brain Exercises for you. Try solving these and submit your full solutions to participate in a draw for \$10 Tim Horton's gift card. You may submit your solutions in the rectangular container labeled 'Problem of the Week' located in MSLC- room AC312 (with your name and email address), or send email to soheekang@utsc.utoronto.ca. Draw winner will be notified through email.

Question 1:

My friend and I are hoping to meet for lunch. We will each arrive at our favorite restaurant at a random time between noon and 1 p.m., stay for 15 minutes, then leave. What is the probability that we will meet each other while at the restaurant? (For example, if I show up at 12:10 and my friend shows up at 12:15, then we'll meet; on the other hand, if I show up at 12:50 and my friend shows up at 12:20, then we'll miss each other.)

It is amazing how the golden ratio captures the proportions of pleasing shapes in art, architecture, geometry and nature. Sea shells, sunflowers, the flight paths of falcons, pineapples' skins, and galaxies all display the beauty of logarithmic spirals--nature's way of expressing the golden ratio.

References:

Livio, M. (2008). *The Golden Ratio: the story of Phi, the world's most astonishing number*. New York: Broadway Books.

Starbird, B. (2013), *The Heart of Mathematics: An Innovation to Effective Thinking*

Question 2:

Fang and Olivia are playing a game. Fang starts with a pile of 1000 pennies, and Olivia starts with a pile of 500 pennies. On each turn, one of them flips a quarter. If the quarter comes up heads, Fang gives Olivia a penny. If the quarter comes up tails, Olivia gives Fang a penny. The game ends when one of them runs out of pennies (it will probably take a while). What is the probability that Fang wins?



The Probability of Love

By Olivia Rennie
Statistics TA, MSLC

Once upon a time, Princess P-value lived in a faraway kingdom, called Statsopia, with her father, King Z-score. Princess P-value was not very happy, though. She was engaged to the not-so-nice Sir Norm Al Distribution, and felt pretty hopeless about where her life was headed.



After one particularly upsetting day, Princess P-value (who went by the nickname "Val") ran down to the river near her castle, needing a spot to be alone to cry about how miserable she was. Suddenly, however, Val slipped on the wet river bank, sending her tumbling helplessly into the raging rapids below.



At that very moment, King Z-score and his crew came running, searching for the lost princess...



All of a sudden, a peasant boy who was out for a stroll came to the rescue, saving Val from an untimely end just in the nick of time!



I suppose you are right... even if he IS just a peasant... hmmm... how about we invite him to join us at the Royal Shapiro-Wilk Ceremony next month?... I believe that would be a fitting reward for his "heroic deed."



And so it was settled. Ollie Outlier, the lowly peasant boy from the local village, would be joining the royal family and their wealthy friends at the prestigious Shapiro-Wilk Ceremony...



Hi Ollie... I... just wanted to thank you very much for saving my life. It was foolish of me to be running so fast beside the river, and I'm very grateful that you were there to prevent me from falling in.



Later that week...

Val and Ollie had indeed fallen in love with each other. However, Val sadly knew they could never be together. Her father would never approve of her marrying someone from a non-normal distribution, especially one as full of outliers as Ollie's...



Unsurprisingly, Norm got a very large p -value, 0.967, suggesting that he did indeed come from a normally distributed population, just as the royal family did (Fail to reject H_0)

And the p -value is... 0.967!



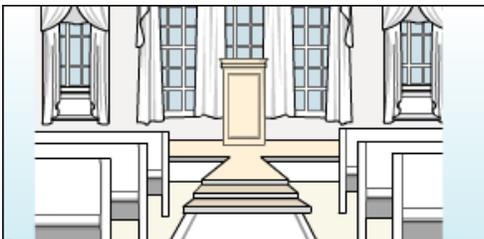
I knew you were one of us, my boy!
Welcome to the family!

There was, however, one thing that the King failed to consider. That was the ancient loophole in the royal rules, known as the **CENTRAL LIMIT THEOREM**. With it, even a non-normal peasant could qualify to marry one of the royalty...

With large enough samples of Ollie's wonderful personality, sense of humour, and kindness, his sampling distribution becomes closer and closer to a normal distribution, regardless of the shape of the original population! This means that those outliers wouldn't make such a big difference after all!



Ollie was quite surprised that such a beautiful princess would even think to come and thank him for what he did, but the two struck up a conversation, and before he knew it, they were already making plans to see each other again...



The day of the Royal Shapiro-Wilk Ceremony finally arrived. This is a special ceremony, designed to test whether the Princess' future fiance comes from a normal distribution, like Princess P -value herself, or not.

This ceremony would essentially decide whether Val would end up having to marry Sir Norm Al Distribution...

Over the next several weeks, Val and Ollie spent more and more time together. After only having her cruel fiancé, Sir Norm Al Distribution, for company, Val was incredibly happy to finally have someone who brought her joy and truly cared about her. For the first time in her life, she was actually happy.



H_0 : Norm comes from a normally distributed population

Ha: Norm does not come from a normally distributed population (AKA Norm is not suited to marry the Princess)

No father!
Please! I don't love Norm. I love Ollie!
Please, test him too! See if he qualifies to marry me instead!

But Val couldn't accept marrying Norm. Not after finding true love with Ollie....

And so, over time, the king got enough samples of Ollie's character to find out that he was actually a pretty normal guy. Certainly normal enough to marry Val. The two became husband and wife, and...



Despite the King's rage, Ollie was tested as well. Sadly, this time H_0 was rejected: all the evidence suggested that Ollie did not come from a normally distributed population, and thus was not, in the King's eyes, fit to marry the Princess.



0.0000000432

LIVED HAPPILY EVER AFTER!!!



THE END!

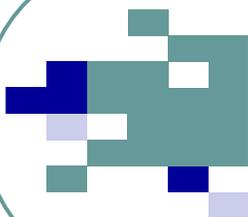
THE WARM AND FUZZIES

By: UTSC Psychology student

As a first year Specialist in Psychology Co-op student, I was dreading the two statistics courses I was going to have to take (PSYB07: Data Analysis in Psychology, and PSYC08: Advanced Data Analysis in Psychology). I had never been the strongest math student in high school. I had barely passed grade 11 math by the skin of my teeth and I made sure to stay well away from grade 12 math. Once second year rolled around, I had to take my two statistics courses. Saying farewell to my GPA, I enrolled in the course.

At first, I was struggling quite a bit with some fairly basic concepts, like boxplots and probability measurements. I went to the MSLC to get some help with the course material and discovered that I had to go back and review some pretty basic ideas from high school to make sure I was able to understand the material in my stats course. After a pretty tough semester—where I often stayed late in the MSLC— I managed to get a pretty decent mark in PSYB07. Not bad, but not too great either. Fortunately, I was a bit more proactive in the next semester when I was taking PSYC08. I did not wait until I was having trouble understanding something to ask for help. I came in to the MSLC to look ahead and prepare for some of the more difficult concepts, like two-way and repeated measure ANOVAs. At the end of another pretty tough semester, I managed to end up with a mark that I was really happy with.

As I entered onto my co-op work term, I managed to get hired at a lab that I found very fascinating. The lab that I had gotten hired into focused on neuroimaging in psychiatric illnesses. I soon found that neuroimaging had some of the most complex statistical analyses I have ever seen. With the help of one of the Master's students, and some more extra hours at the MSLC, I was able to learn how to use statistical software such as R and SAS. Now I am fully comfortable with statistical analyses, and I am actually helping teach other students in the concepts and computational analysis of statistics.

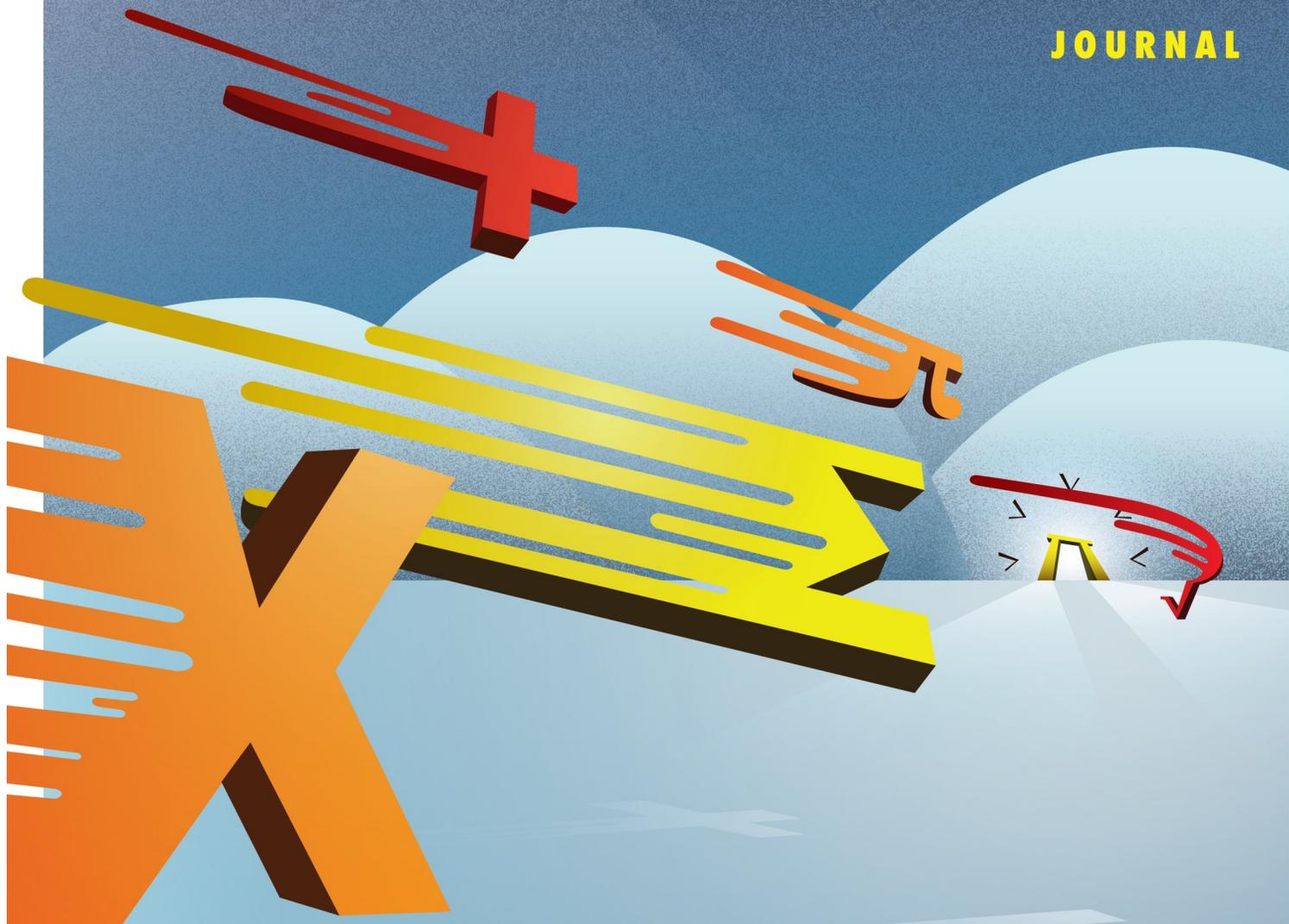


Note on images and illustrations:

Milky Way image on pg. 10 taken from NASA website
 Probability of Love cartoon on pg.12 created on Pixton.com
 Image on pg.14 taken from MSLC database

MATH IN ACTION

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Math In Action is an undergraduate mathematical research video journal. The journal aims to provide a platform for mathematics undergraduates to share their work with peers and academics alike. Submissions are accepted in form of video presentation, along with a two page extended abstract. All forms of creativity are strongly encouraged.

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CONTRIBUTORS

EDITOR-IN CHIEF **ZOHREH SHAHBAZI**

STATISTICS REVIEWER **SOHEE KANG**

DESIGNER **MANAAL HUSSAIN**

PUBLISHING EDITOR **TREVOR CAMERON**

COMMUNICATION OFFICER **OLIVIA RENNIE**

ILLUSTRATOR **MICHAEL GAYLE**

SPECIAL CONTRIBUTIONS **RAYMOND GRINNELL**

JIAHENG LI

BATUL FATIMA

